[2] $\boxed{2r^2-4r+1=0}$ $\frac{1}{2}$

$$r=\frac{4\pm\sqrt{16-8}}{4}=\frac{4\pm2\sqrt{2}}{4}=\boxed{\frac{2\pm\sqrt{2}}{2}}$$ $\frac{1}{2}$

$$\boxed{x=c_1 t^{\frac{2+\sqrt{2}}{2}}+c_2 t^{\frac{2-\sqrt{2}}{2}}}$$ 1

[3] $\boxed{y_2 = ve^{-3x}}$ $\frac{1}{2}$

$\boxed{y_2' = v'e^{-3x} - 3ve^{-3x}}$ $\frac{1}{2}$

$$y_2'' = v''e^{-3x} - 3v'e^{-3x} - 3v'e^{-3x} + 9ve^{-3x}$$

$$= v''e^{-3x} - 6v'e^{-3x} + 9ve^{-3x} \quad \tfrac{1}{2}$$

$$\begin{aligned} xy_2'' + (6x+1)y_2' + (9x+3)y_2 &= e^{-3x}\big[xv'' - 6xv' + 9xv + (6x+1)v' + (-18x-3)v + (9x+3)v\big] \quad 1 \\ &= e^{-3x}\big[xv'' + v'\big] \end{aligned}$$

CHECKPOINT: NO v WITHOUT ' $\frac{1}{2}$

$$= 0$$

LET $u = v' \longrightarrow u' = v''$

$$xu' + u = 0$$

$$x\frac{du}{dx} = -u$$

$\int \frac{1}{u}\,du = \int \frac{1}{x}\,dx$ CHECKPOINT: SEPARABLE $\frac{1}{2}$

$$\ln|u| = -\ln|x|$$

$$v' = \boxed{u = x^{-1}} \quad \tfrac{1}{2}$$

$$v = \ln|x|$$

$$y = \underbrace{\boxed{c_1e^{-3x}}}_{\frac{1}{2}} + \boxed{c_2e^{-3x}\ln|x|} \quad \tfrac{1}{2}$$

[4] $r^2+4r+13=0$

$(r+2)^2+9=0$

$r=-2\pm 3i$ ½

$y=c_1e^{-2x}\cos 3x+c_2e^{-2x}\sin 3x$ ①

$y(0)=c_1=4$ ½

$y'=-8e^{-2x}\cos 3x-12e^{-2x}\sin 3x$
$+3c_2e^{-2x}\cos 3x-2c_2e^{-2x}\sin 3x$ ①

$y'(0)=-8+3c_2=-5 \longrightarrow c_2=1$ ½

$y=4e^{-2x}\cos 3x+e^{-2x}\sin 3x$ ½

[5] [a] $(r-(a+bi))(r-(a-bi)) = \boxed{r^2 - 2ar + a^2 + b^2}$ FROM LECTURE $\frac{1}{2}$

[b] $\boxed{x^2 y'' + (-2a+1)xy' + (a^2+b^2)y = 0}$ 1

[c] $y = x^a \cos(b \ln x)$

$y' = \boxed{a x^{a-1} \cos(b\ln x) + x^a(-\sin(b \ln x))\frac{b}{x}}$ 1

$= a x^{a-1}\cos(b \ln x) - b x^{a-1} \sin(b \ln x)$

$y'' = \boxed{a(a-1)x^{a-2}\cos(b\ln x) + a x^{a-1}(-\sin(b\ln x))\frac{b}{x} - b x^{a-1}\cos(b \ln x)\frac{b}{x} - b(a-1)x^{a-2}\sin(b\ln x)}$ $1\frac{1}{2}$

$\frac{1}{2}$ $= \boxed{(a^2-a-b^2)x^{a-2}\cos(b\ln x) + (b-2ab)x^{a-2}\sin(b\ln x)}$

$$\begin{aligned} x^2y'' \\ +(-2a+1)xy' \\ +(a^2+b^2)y \end{aligned} = \boxed{\begin{aligned} &x^a\left[(\cancel{a^2}-\cancel{a}-\cancel{b^2})\cos(b\ln x) + (\cancel{b}-\cancel{2ab})\sin(b\ln x)\right. \\ &\quad +(-\cancel{2a^2}+\cancel{a})\cos(b\ln x) + (\cancel{2ab}-\cancel{b})\sin(b\ln x) \\ &\quad \left.+(\cancel{a^2}+\cancel{b^2})\cos(b\ln x)\right] \\ &= 0 \end{aligned}}$$

$1\frac{1}{2}$

[d] $$\begin{vmatrix} x^a\cos(b\ln x) & x^a\sin(b\ln x) \\ ax^{a-1}\cos(b\ln x)-bx^{a-1}\sin(b\ln x) & ax^{a-1}\sin(b\ln x)+bx^{a-1}\cos(b\ln x) \end{vmatrix}$$

1

$$= ax^{2a-1}\cancel{\cos(b\ln x)}\sin(b\ln x)+bx^{2a-1}\cos^2(b\ln x)$$
$$-ax^{2a-1}\cancel{\cos(b\ln x)}\sin(b\ln x)+bx^{2a-1}\sin^2(b\ln x)$$

1

$$= bx^{2a-1}\left(\cos^2(b\ln x)+\sin^2(b\ln x)\right)$$

$$= \boxed{bx^{2a-1}}\ \boxed{\neq 0}$$ $\frac{1}{2}$

$\frac{1}{2}$